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Applicability of Unimodal Stochastic Approaches in Simulating Flow in Bimodal Heterogeneous Formations

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ABSTRACT

Most existing stochastic models assume that the porous medium being studied can be characterized by one single correlation scale. However, hydraulic properties exhibit spatial variations at various scales, thus stochastic models developed for unimodal media may not be applicable to flow and transport in a bimodal heterogeneous medium. The aim of this study is to investigate under what circumstances the second-order moment-based stochastic models are applicable to the bimodal porous medium. We assume that two materials (categories) in the porous medium may have a different mean, variance, and correlation scale. The distribution of materials in the domain is characterized by indicator random functions. We derived expressions for the covariance of the indicator random variables and that of the composite field in terms of categorical proportions and transition probability. We solved the second-order flow moment equations for the "equivalent" unimodal field with an exponential covariance of a single correlation scale computed for the composite field. On the other hand, we conduct two sets of Monte Carlo simulations: one with bimodal random fields and the other with equivalent unimodal fields. Numerical experiments show that a bimodal $\ln K$ field may be well approximated with an equivalent unimodal field when the bimodal distribution is highly asymmetric, under which condition the applicability of the second-order moment-based stochastic model is subject to the same limitation of relatively small variances as that for unimodal fields. When the bimodal distribution is symmetric, although it cannot be adequately represented by an equivalent unimodal distribution the second-order moment-based stochastic model seems to be applicable to larger composite variance systems than it does for an asymmetric distribution.

1. Introduction

It is well known that geological formations are ubiquitously heterogeneous. Stochastic approaches to flow and transport in heterogeneous porous media have been extensively studied in the last two decades and many stochastic models have been developed [e.g., *Dagan*, 1989; *Gelhar*, 1993; *Zhang*, 2001]. Most of these models assume that the porous medium being studied can be characterized by one single correlation scale on the basis of some field studies [*Hoeksema and Kitanidis*, 1984; *Gelhar*, 1993], which have showed that the hydraulic conductivity in some cases is unimodal. However, this in general may not be true. In fact, hydraulic properties exhibit spatial variations at various scales, such as at the laboratory scale due to variations in pore geometry, at the field scale due to soil stratifications, and at the regional scale due to large-scale geological variability.

Stochastic models developed for unimodal heterogeneous porous media may not be directly applicable to flow and transport in bimodal heterogeneous porous media. One question we should ask is under what circumstances the stochastic models developed mainly for the unimodal heterogeneous media can be applied to flow in the bimodal heterogeneous systems.

A few studies have been conducted on flow and transport in a bimodal porous medium [*Desbarats*, 1987, 1990; *Rubin and Journel*, 1991; *Rubin*, 1995; *Russo et al.*, 2001]. *Desbarats* [1987, 1990] modeled the permeability of a sandstone reservoir as a bimodal attribute of two possible values, K_{ss} and K_{sh}. Variations within sandstone or shale were ignored. *Rubin and Journel* [1991] decomposed the random function of interest, say, $Z(x)$, into a series of indicator random functions, allowing to assign specific spatial structure to each class of Z values. The effect of bimodal heterogeneity on transport has been studied by *Rubin and Journel* [1991] and *Rubin* [1995]. Both assumed that the spatial distribution of $Z_1(x)$ and $Z_2(x)$ are independent of the indicator random function, and thus the spatial structures of Z_1, Z_2, and the indicator random function could be assigned arbitrarily. Recently, *Russo et al.* [2001] investigated flow and transport of a tracer solute in variably saturated bimodal heterogeneous porous media and the corresponding "equivalent" unimodal media. Again, the indicator function (or variable) is modeled by assigning its own spatial structure, independent of the properties of the composite materials. It is known that the mean and variance of the indicator variable are related to the volumetric proportion of each individual material. For a porous medium with two materials, for example, $\langle I_i(\boldsymbol{x}) \rangle = p_i$ and $\sigma_{I_i}^2 = p_1 p_2$, where p_i and $I_i(\boldsymbol{x})$ are the volumetric proportion of material i and its indicator variable. Therefore, it may not be reasonable to assume that the correlation structures of the indicator variables can be assigned arbitrarily. The second question we would like to ask is how to quantitatively determine the spatial structure of the indicator random function based on the proportions and spatial structures of the two categories.

The Markov chain method has been applied to geological formations with different materials [*Harbaugh and Bonham-Carter*, 1970; *Lin and Harbaugh*, 1984; *Carle and Fogg*, 1996; *Carle and Fogg*, 1997]. The distribution of materials is characterized by the transition probability between different materials. It has been shown [*Carle*, 1996; *Carle* and *Fogg*, 1996] that, in characterizing the structure of the indicator random functions, the transition probability between different categories is equivalent to the covariance of the indicator random functions and the former can be easily derived from field measurements.

The aims of this study are (1) to derive explicit expressions for the covariance function of indicator random functions, based on the properties of different materials; and (2) to discuss the general requirements at which the second-order moment-based stochastic model with a covariance function of a single correlation length may be applied to a porous medium with bimodal heterogeneity.

2. Mean and covariance of bimodal heterogeneous fields

Let $Y(\boldsymbol{x})$ be an attribute of interest, such as log hydraulic conductivity, and be expressed as

$$Y(\boldsymbol{x}) = I_1(\boldsymbol{x})Y_1(\boldsymbol{x}) + I_2(\boldsymbol{x})Y_2(\boldsymbol{x}) \tag{1}$$

where $Y_i(\boldsymbol{x})$, $i = 1, 2$, stands for different types (e.g., facies) of the same attribute $Y(\boldsymbol{x})$ at location $\boldsymbol{x}$, and $I_i(\boldsymbol{x})$ are indicator spatial random functions defined over a domain Ω as $I_i(\boldsymbol{x}) = 1$ if category i occurs at location $\boldsymbol{x}$, and $I_i(\boldsymbol{x}) = 0$ otherwise. For a continuous attribute $Y(\boldsymbol{x})$, $I_i(\boldsymbol{x})$ can be defined using a set of different cutoffs [*Deutsch and Journel*, 1998; *Rubin and Journel*, 1991]. It is clear that $I_1(\boldsymbol{x}) + I_2(\boldsymbol{x}) = 1$ for any $\boldsymbol{x} \in \Omega$. By definition, the joint probability of $I_i(\boldsymbol{x})$ and $I_j(\boldsymbol{x})$ can be expressed as

$$p_{ij}(\boldsymbol{x}, \boldsymbol{\chi}) = \Pr\{I_i(\boldsymbol{x}) = 1, I_j(\boldsymbol{\chi}) = 1\} = E\{I_i(\boldsymbol{x})I_j(\boldsymbol{\chi})\} \tag{2}$$

and their marginal probability as

$$p_i(\boldsymbol{x}) = \Pr\{I_i(\boldsymbol{x}) = 1\} = E\{I_i(\boldsymbol{x})\} \tag{3}$$

The transition probability $t_{ij}(\boldsymbol{x},\boldsymbol{\chi})$ is defined as the probability of category j occurring at location $\boldsymbol{\chi}$, given the condition that category i occurs at location $\boldsymbol{x}$:

$$t_{ij}(\boldsymbol{x}, \boldsymbol{\chi}) = \Pr\{I_j(\boldsymbol{\chi}) = 1 \mid I_i(\boldsymbol{x}) = 1\} = E\{I_i(\boldsymbol{x})I_j(\boldsymbol{\chi})\} / E\{I_i(\boldsymbol{x})\} \tag{4}$$

The covariance of the indicator functions can be given as

$$C_{I,ij}(\boldsymbol{x}, \boldsymbol{\chi}) = E\{I_i(\boldsymbol{x})I_j(\boldsymbol{\chi})\} - E\{I_i(\boldsymbol{x})\}E\{I_j(\boldsymbol{\chi})\} \tag{5}$$

Substituting (3) and (4) into (5), the covariance of indicator random variables can be expressed in terms of categorical proportions and the transition probability $t_{ij}(\boldsymbol{x},\boldsymbol{\chi})$

$$C_{I,ij}(\mathbf{x}, \chi) = [t_{ij}(\mathbf{x}, \chi) - p_j(\chi)] p_i(\mathbf{x}) \tag{6}$$

It should be noted that, since $t_{ij}(\boldsymbol{x}, \boldsymbol{\chi})p_i(\boldsymbol{x}) = t_{ji}(\boldsymbol{\chi}, \boldsymbol{x})p_j(\boldsymbol{\chi})$, the covariance of the indicator random function is symmetric with respect to positions $\boldsymbol{x}$ and $\boldsymbol{\chi}$, but the transition probability $t_{ij}(\boldsymbol{x}, \boldsymbol{\chi})$ is not symmetric. It has been argued [e.g., *Rubin and Journel*, 1991; *Rubin*, 1995] that $I_i(\boldsymbol{x})$ and $Y_j(\boldsymbol{x})$ should be mutually uncorrelated. Under this condition and with the additional assumption of stationarity, the mean and covariance of Y can be derived as

$$\langle Y \rangle = p_1 \langle Y_1 \rangle + p_2 \langle Y_2 \rangle \tag{7}$$

$$C_Y(h) = \sum_{i,j=1}^{2} [C_{I,ij}(h) + p_i p_j] C_{Y,ij}(h) + \sum_{i,j=1}^{2} C_{I,ij}(h) \langle Y_i \rangle \langle Y_j \rangle \tag{8}$$

$$\sigma_Y^2 = p_1 \sigma_{Y_1}^2 + p_2 \sigma_{Y_2}^2 + p_1 p_2 \left(\langle Y_1 \rangle - \langle Y_2 \rangle \right)^2 \tag{9}$$

where

$$C_{I,ij}(h) = [t_{ij}(h) - p_j] p_i \tag{10}$$

and h is the separation distance.

3. Covariance of indicator random functions

In the last section, we expressed the covariance of the composite log hydraulic conductivity field in terms of the covariance functions of the indicator random functions. It is commonly assumed in the literature that the correlation structures of the indicator random variables be independent of the individual categories [*Rubin and Journel*, 1991; *Rubin*, 1995; *Russo et al.*, 2001]. In this section, we derive expressions for the covariance of the indicator random functions using the Markovian chain approach, based on the statistics of different categories in a porous medium.

It is assumed in a three-dimensional Markovian chain model that spatial variability in any direction can be characterized by a one-dimensional Markovian chain model [*Lin and Harbaugh*, 1984]. For a one-dimensional Markovian chain model, the continuous-lag transition probability matrix T for any lag h can be written as [*Carle* 1996; *Carle and Fogg*, 1996, 1997]:

$$T(h) = e^{Rh} \tag{11}$$

where R is a 2×2 transition rate matrix whose entry r_{ij} represents the rate of change from category i to category j per unit length of category i in the given direction. If the transition rate matrix R is known, the transition probability matrix T can be evaluated by the eigenvalue analysis. Let η_i, $i =1,2$, be eigenvalues of the transition rate matrix R, and Z_i, $i =1, 2$, be their corresponding spectral component matrices, which are evaluated by

$$Z_i = \prod_{m \neq i}(\eta_m E - R) \bigg/ \prod_{m \neq i}(\eta_m - \eta_i) \tag{12}$$

where E is the identical matrix,. Then (11) becomes

$$T(h) = e^{\eta_1 h} Z_1 + e^{\eta_2 h} Z_2 \tag{13}$$

Now we focus on how to evaluate R and relate the covariance of the indicator random functions to the statistics of different materials in a porous medium. Taking derivative of (11) with respect to h and let $h = 0$, we have

$$R = \frac{dT(h)}{dh}\bigg|_{h=0} \tag{14}$$

Note that the transition probability t_{ij} has to satisfy $t_{i1} + t_{i2} = 1$, $i = 1, 2$, and $p_1 t_{1j} + p_2 t_{2j} = p_j$, $j = 1,2$. It follows immediately that transition rate r_{ij} satisfies

$$r_{i1} + r_{i2} = 0 \qquad\qquad i =1,2 \tag{15}$$
$$p_1 r_{1j} + p_2 r_{2j} = 0 \qquad\qquad j=1, 2 \tag{16}$$

Equations (15) and (16) imply that $det(R) = 0$, therefore, one of two eigenvalues of R, say η_1, must be zero. *Carle* [1996] showed that the diagonal terms of the transition rate matrix R is related to mean lengths:

$$r_{ii} = -\frac{1}{L_i} \tag{17}$$

From (15), if the mean lengths in a given direction are L_1 and L_2, respectively, the transition rate matrix must be in the following form

$$R = \begin{bmatrix} -1/L_1 & 1/L_1 \\ 1/L_2 & -1/L_2 \end{bmatrix} \tag{18}$$

Two eigenvalues are $\eta_1 = 0$ and $\eta_2 = -1/p_2L_1 = -1/p_1L_2$, and their corresponding spectral matrices are

$$Z_1 = \begin{bmatrix} p_1 & p_2 \\ p_1 & p_2 \end{bmatrix} \qquad Z_2 = \begin{bmatrix} p_2 & -p_2 \\ -p_1 & p_1 \end{bmatrix} \tag{19}$$

From (13), the transition probability t_{ij} can be written as

$$t_{ij}(h) = p_j + (\delta_{ij} - p_j)e^{-h/\lambda_I} \tag{20}$$

where the parameter λ_I is defined as

$$\lambda_I = p_1 L_2 = p_2 L_1 = L_1 L_2 /(L_1 + L_2) \tag{21}$$

Substituting (20) into (10), one obtains the covariance of the indicator functions

$$C_{I,ij}(h) = p_i(\delta_{ij} - p_j)e^{-h/\lambda_I} \tag{23}$$

It is seen that λ_I in fact is the correlation length of the indicator functions and is not independent of the spatial structures of the two categories. It should be emphasized that if the mean lengths depend on directions, the correlation length λ_I is also direction-dependent. Since for a stationary field, the categorical proportions of the field are the same in any direction. Therefore, it is seen from (21) that the isotropic ratio between any two directions equals to the ratio of the mean lengths of a category in these two directions. Once the covariances of the indicator random functions $C_{I,ij}(h)$ are known, $C_Y(h)$ can be calculated using (8) and the integral scale of the Y field can be derived by integrating $C_Y(h)$:

$$C_Y(h) = p_1^2 \sigma_{Y_1}^2 e^{-\frac{h}{\lambda_1}} + p_2^2 \sigma_{Y_2}^2 e^{-\frac{h}{\lambda_2}} + p_1 p_2 (\langle Y_1 \rangle - \langle Y_2 \rangle)^2 e^{-\frac{h}{\lambda_I}} + p_1 p_2 e^{-\frac{h}{\lambda_I}} (\sigma_{Y_1}^2 e^{-\frac{h}{\lambda_1}} + \sigma_{Y_2}^2 e^{-\frac{h}{\lambda_2}}) \tag{24}$$

$$\lambda_Y = \frac{p_1 \sigma_{Y_1}^2 \lambda_1 + p_2 \sigma_{Y_2}^2 \lambda_2 + p_1 p_2 (\langle Y_1 \rangle - \langle Y_2 \rangle)^2 \lambda_I + p_1 p_2 \sigma_{Y_1}^2 \frac{\lambda_1 \lambda_I}{\lambda_1 + \lambda_I} + p_1 p_2 \sigma_{Y_2}^2 \frac{\lambda_2 \lambda_I}{\lambda_2 + \lambda_I}}{p_1 \sigma_{Y_1}^2 + p_2 \sigma_{Y_2}^2 + p_1 p_2 (\langle Y_1 \rangle - \langle Y_2 \rangle)^2} \tag{25}$$

4. Numerical Implementation

To investigate the applicability of the unimodal stochastic models with a single

correlation length to flow in a bimodal system, in this section we illustrate a few examples for flow in a two-dimensional horizontal, saturated porous medium with two materials. The square domain of 12m × 12m is uniformly discretized into 60 × 60 square elements with a size of 0.2m × 0.2m. Two factors have been considered in specifying material properties for the cases studied: the symmetry of the $Y(\boldsymbol{x})$ distribution and the contrast of between $\langle Y_1(\boldsymbol{x})\rangle$ and $\langle Y_2(\boldsymbol{x})\rangle$. Detail specifications for materials in each case are listed in Table 1. The correlation length of the indicator function, λ_I, is specified according to (21). It is assumed that the covariance of $Y_i(\boldsymbol{x})$ is exponential with a range $l_i = 3\lambda_i \leq L_i$. The values of $\langle Y_1\rangle$ and $\langle Y_2\rangle$ are chosen such that $\langle Y\rangle$ of the composite field is zero. The parameters in the right part of the table (the last three columns), i.e., $\langle Y\rangle$, σ_Y^2, and λ, are the means (weighted), variances and correlation lengths of the composite field that are calculated using (7), (9), and (25).

In all cases, the hydraulic head is prescribed at the left and right boundaries as 10.5 m and 10.0 m, respectively, which produces a mean flow from the left to the right. The lateral boundaries are prescribed as no-flow boundaries.

The purpose of designing these cases is briefly described as follows. Cases 1-2 (p_1 = 0.3) are designed against Cases 3-4 (p_1 = 0.5) to test the effects of categorical proportions and symmetry of the Y distribution. The pair of Case 1 and Case 2 (or, Case 3 and Case 4) is compared to explore the effect of the contrast of the mean Y between two materials.

Table 1. Parameter specifications for all cases

Parameter	p_1	$\langle Y_1\rangle$	$\langle Y_2\rangle$	$\sigma_{Y_1}^2$	$\sigma_{Y_2}^2$	λ_i	$\langle Y\rangle$	σ_Y^2	λ
Case 1	0.3	-1.4	0.6	0.1	0.1	1.2	0.0	0.94	2.362
Case 2	0.3	-2.8	1.2	0.1	0.1	1.2	0.0	3.46	2.477
Case 3	0.5	-1.0	1.0	0.1	0.1	1.2	0.0	1.10	1.724
Case 4	0.5	-2.0	2.0	0.1	0.1	1.2	0.0	4.10	1.780

For each case, we conduct two sets of Monte Carlo simulations and compare results against those from the second-order moment-based stochastic models [*Zhang and Winter*, 1999; *Zhang and Lu*, 2001]. The first set of Monte Carlo simulations is performed for flow in a porous medium with two materials. For this purpose, we first generate 5,000 two-dimensional 61 × 61 ($Y(\boldsymbol{x})$ is defined at nodal points) Markovian chain random fields of two categories with given proportions p_1 and p_2, using Transition Probability Geostatistical Software (T-PROGS) developed by *Carel* [*Carel*, 1996]. We then generate two sets of 5,000 two-dimensional 61 × 61 unconditional Gaussian realizations with zero mean, unit variance, and an exponential covariance function with a correlation length $\lambda_1 = \lambda_2 =$ 1.20 m as specified in Table 1, using a sequential Gaussian random field generator *sgsim* from GSLIB [*Deutsch and Journel*, 1998]. The quality of these Gaussian random fields is first checked by comparing the sample mean and variance of these unconditional realizations with the specified zero mean and unit variance. The variogram calculated from generated Gaussian realizations is also compared with the analytical exponential model. These comparisons indicate that the generated Gaussian realizations satisfy the specified mean, variance, and correlation length very well. Each Markovian

chain field is then combined with two continuous Gaussian realizations that are scaled from zero mean and unit variance to the specified means and variances of the two categories to form a new random field Y. As an example, one bimodal Markovian chain (indicator, geometry) realization with proportion $p_1 = 0.3$ and the combined bimodal realization for Case 2 are shown in Figure 1, with a weighted $\langle Y \rangle = 0.0$ and a total variance $\sigma_Y^2 = 3.46$. The histograms of the generated log hydraulic conductivity realizations for all cases are depicted in Figure 2, where each histogram is obtained from 18,605,000 (= 61 × 61 × 5,000) data points. For each realization, the saturated steady-state flow equation is solved using Finite-Element Heat- and Mass-Transfer code (FEHM) developed by *Zyvoloski et al.* [1997]. The convergence of Monte Carlo simulations is checked by selecting some points in the domain and plotting the sample statistics of head and fluxes at these points against the number of Monte Carlo simulations (not shown). It is found that 5,000 realizations are adequate for all these cases. The second set of Monte Carlo simulations is done for the equivalent unimodal fields with an exponential covariance. For each case, the mean, variance, and correlation scale are computed from the composite fields with (7), (9), and (25), respectively. The first set of Monte Carlo simulations is termed "bimodal Monte Carlo", and the second called "unimodal Monte Carlo". The bimodal Monte Carlo simulation results are considered the "true" solution that is the basis for comparisons between different approaches.
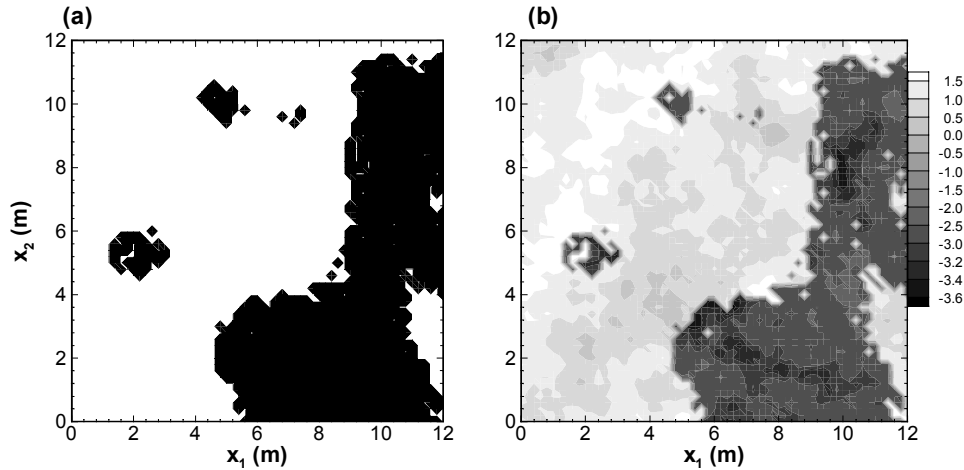

Figure 1. Examples of a Markovian (indicator, geometry) realization (a) and a composite log hydraulic conductivity field (b) using the Markovian realization and two Gaussian realizations.

The second-order moment-based stochastic model [*Zhang and Winter*, 1999; *Zhang and Lu*, 2001] is applied to a statistically homogeneous field with a mean calculated from (7) and an exponential covariance function with the variance and the single correlation length computed from (9) and (25). For the purpose of comparison, we also apply the second-order moment-based stochastic model to the field with the mean $\langle Y \rangle$ and bimodal covariance C_Y computed from (7) and (24), respectively. We call the former the unimodal moment-based approach, and the latter the bimodal moment-based approach. The main

purpose of this study is to discuss the applicability of the unimodal moment-based and unimodal Monte Carlo approaches to flow in a bimodal porous medium. It is also of interest to see if the second-order bimodal moment-based stochastic model will make any improvement, comparing to the second-order unimodal moment-based stochastic model.

5. Results and Discussion

For our cases with prescribed heads at the left and right boundaries, though there are only slight differences on the mean head computed using different approaches (not shown), the variances associated with the mean head predictions could, however, be significantly different. Figure 3 illustrates the head variance profiles along a line passing through the center of the flow domain and parallel to the x_1 direction, from the four different approaches. The solid lines in the figure stand for the head variance computed from the bimodal Monte Carlo simulations (the "true" solution), dashed lines for results from the bimodal moment-based stochastic model, open circles for the unimodal Monte Carlo simulations, and open squares for the unimodal moment-based stochastic model. It is seen that for each case considered the head variance obtained from the bimodal moment-based stochastic model is essentially the same as that computed from the unimodal moment-based stochastic model. This is understandable as the covariance (24) of the composite log hydraulic conductivity field can be well approximated by an exponential covariance of a single correlation length.

From the figure it is seen that, when the total variance of the composite bimodal field is small (for example, $\sigma_Y^2 = 0.94$ and 1.10, for Case 1 and Case 3, respectively), the head variances computed from the four different approaches are very close (Figs. 3a and c), even though the distribution of log hydraulic conductivity is bimodal and/or asymmetric (Case 1).

When the total variance of the composite bimodal field is increased by increasing the contrast between two materials, the head variance profiles show a very interesting pattern. The results from the two sets of Monte Carlo simulations are in a very close agreement for Case 2 whereas the two sets of Monte Carlo results differ significantly for Case 4. This indicates that the bimodal random field with $p_1 = 0.3$ (i.e., 30-70 proportions) in Case 2 is very well represented by an equivalent unimodal field with an exponential covariance while the bimodal field with $p_1 = 0.5$ (50-50 proportions) in Case 4 may not be adequately represented by an equivalent unimodal field. It is understandable after we realize that in Case 4 there are two distinct, equally important modals while in Case 2 one of the two modals dominates. Comparing the second-order moment-based approaches with the Monte Carlo simulations in Case 2 reveals that there exists a large difference, which may be attributed to the effects of higher-order terms truncated in the second-order moment-based approaches. However, for Case 4 the agreement between the moment-based approaches and the bimodal Monte Carlo simulations is excellent. This observation is surprising in that the composite variance of log hydraulic conductivity is larger in Case 4 than that in Case 2. At present, we do not have a firm explanation for it. We suspect that this unexpectedly good agreement is related to the fact that the bimodal log hydraulic conductivity distribution is symmetric and the variability in each category is small although the composite one is large, under which circumstance the higher-order terms may be small. We investigate this hypothesis via running more cases (not shown)

of increased composite variances by increasing the contrasts in the means of the categories as well as the variabilities of them and find a consistent pattern that for the cases of symmetric bimodal distributions the agreement between the moment-based approaches and the bimodal Monte Carlo simulations deteriorates as the increase of the composite variance. To validate or invalidate this hypothesis, however, requires an evaluation of high-order terms truncated in the second-order moment-based model, perhaps, on the basis of Monte Carlo simulations.
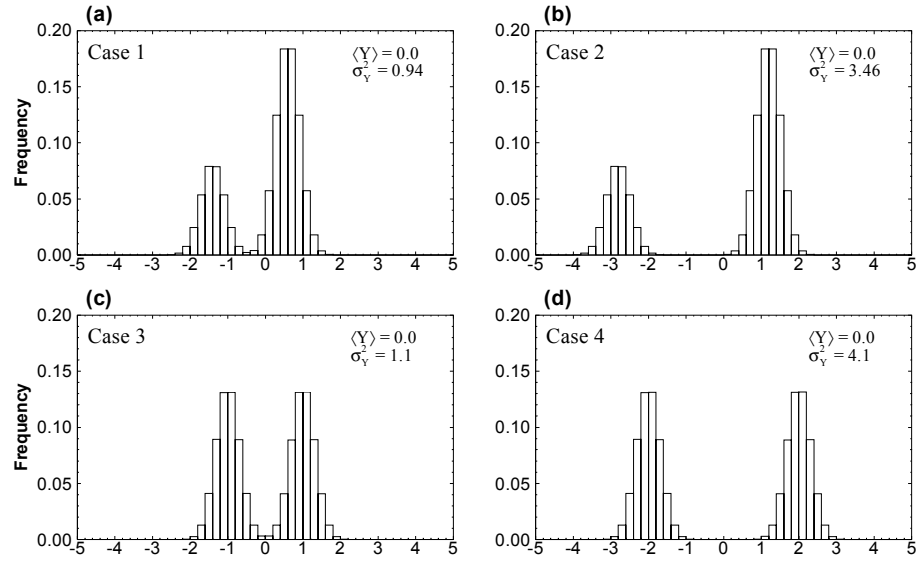

Figure 2. Histograms of the composite log hydraulic conductivity fields for numerical simulation cases.
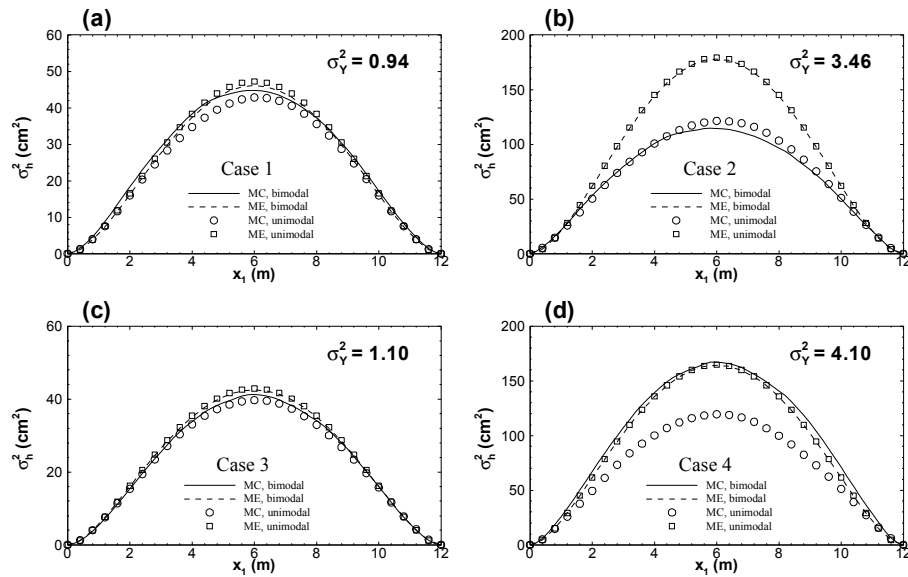

Figure 3. Comparison of the head variance obtained from four different approaches for all cases with prescribed head boundaries.

6. Conclusions

In this study, we considered a porous medium composed of two materials, each of which may have different means, variances and correlation scales of log hydraulic conductivity $Y = \ln K$. We first derived an expression for the correlation length of indicator random functions in terms of the statistics of the two materials in the porous medium, which has been assumed to be independent of the statistics of the two materials in the previous studies [*Rubin and Journel*, 1991; *Rubin*, 1995; *Russo et al.*, 2001]. We then solved stochastic moment equations for flow in an "equivalent" unimodal porous medium using an exponential covariance with a correlation length computed for the composite Y field, compared these results with those from Monte Carlo simulations conducted for flow in the bimodal porous medium and in equivalent unimodal medium, and discussed the applicability of the stochastic moment-based models developed for unimodal porous media to flow in bimodal porous media. We also solved stochastic moment equations for flow in the bimodal medium with covariance computed using (24), instead of a single correlation scale.

Our results show that, when the total variance of the $\ln K$ is small, for example $\sigma_Y^2 < 1.0$, no matter what the $\ln K$ distribution is, the mean head and its variance from both the unimodal and bimodal moment-based stochastic models as well as those from the unimodal Monte Carlo simulation are close to the bimodal Monte Carlo simulation results. This indicates that the unimodal moment-based stochastic models are applicable to a bimodal system in this case.

When the total variance the $\ln K$ is large, the symmetry of the $\ln K$ distribution has a great effect on the applicability of the stochastic moment approaches. When the $\ln K$ distribution is symmetric, results from both unimodal and bimodal moment-based models are very close to those from bimodal Monte Carlo simulations for the composite variance as large as 4.0. As the composite variance increases, the agreement between the second-order moment-based models and the bimodal Monte Carlo simulations deteriorates. When the $\ln K$ distribution is asymmetric, results from two sets of Monte Carlo simulations may be in close agreements, due to the fact that one material dominates the distribution and thus the bimodal porous medium may be well represented by its equivalent unimodal porous medium. For this case and when the composite variance is about 3.5, the results from both unimodal and bimodal moment-based models are significantly different than those from the corresponding Monte Carlo simulations, indicating that the second-order moment-based stochastic models do not work for a relatively large composite variance under these conditions. The applicability of the second-order moment-based stochastic models to a bimodal porous medium is further limited by the fact that in realty the mean log hydraulic conductivities between different materials may differ by several orders of magnitude, thus the composite variance of the $\ln K$ may be very large.

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